

NEW APPROACH TO ^4He CHARGE DISTRIBUTION

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Abstract

We present a study of the ^4He charge distribution based on realistic nucleonic wave functions and incorporation of the nucleons' quark substructure. The central depression of the proton point density seen in modern four-body calculations is too small by itself to lead to a correct description of the charge distribution. We utilize six-quark structures calculated in the Chromodielectric Model for N-N interactions, and we find a swelling of the proton charge distribution as the internucleon distance decreases. These charge distributions are combined with the ^4He wave function using the Independent Pair Approximation and two-body distributions generated from Green's Function Monte Carlo calculations. We obtain a reasonably good fit to the experimental charge distribution without including meson exchange currents.

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I. INTRODUCTION

The charge distribution of nuclei has been the subject of experimental studies for more than forty years. Electron scattering and muonic atoms now provide detailed descriptions of the full range of stable, and many unstable nuclides. Unique among the nuclides are the isotopes ^3He and ^4He because they exhibit a central density about twice that of any other nuclei. There is a long-standing apparent discrepancy between the experimentally extracted charge distributions and detailed theoretical structure calculations which include only nucleon degrees of freedom.

McCarthy, Sick and Whitney [1,2] performed electron scattering experiments on these isotopes up to momentum transfers of 4.5 fm^{-1} yielding a spatial resolution of 0.3 fm . They extracted a “model independent” charge distribution, which means that their analysis of the data is not based upon any assumed functional form for the charge distributions. Their results are shown in Fig. 1. Taken alone, they do not appear to be extraordinary. However, using the experimentally measured proton form factor, which has an rms radius of about 0.83 fm , they unfolded the proton structure from the charge distributions to obtain proton point distributions. For both isotopes there is a significant central depression of about 30% extending to about 0.8 fm . Sick [2] also obtained results where relativistic and meson effects are included. These are shown for ^4He in Fig. 2. One note of caution here is that it is not possible to subtract these effects from the experimental data in a completely model-independent way.

One might assume that such a central depression is to be expected because of the short-range repulsion of the nucleon-nucleon interaction. This is not borne out by numerous detailed theoretical calculations (see, for example, Ref. [3]) none of which finds a *significant* central depression, certainly not of the above magnitude. Relatively smaller central depressions are found in Green’s Function Monte Carlo (GFMC) calculations of ^4He for realistic models of the two- and three-nucleon interaction [4,5].

The status of theoretical structure calculations through mass number 4 is very satisfactory at present. Given any assumed interaction, the few body problem can be solved to within tenths of an MeV in energy and the wave function can be calculated to a precision better than that required for the present discussion.

In using a nucleonic wave function to construct a charge distribution, one must use an assumed nucleon charge density and the possibility of meson exchange contributions. While the meson exchange contributions in the transverse channel are well-constrained (at least at moderate momentum transfer) by current conservation, no such constraint is available in the longitudinal channel. Indeed, meson exchange contributions are of relativistic order and hence one must be careful when interpreting them with non-relativistic wave functions.

Given these caveats, it is possible to reproduce reasonably well the longitudinal form factors of three- and four-body nuclei within a nucleon-plus-meson-exchange model [4–6]. The current and charge operators are constructed from the N-N interaction and required to satisfy current conservation at non-relativistic order. The resulting meson-nucleon form factors are quite hard, essentially point-like [4,5]. This raises the possibility of explaining the form factors in quark or soliton based models, which would describe the short-range two-body structure of the nucleons in a more direct way than is available through meson exchange current models. See, for example, the model by Kisslinger et al. [7].

We present here a possible explanation of the electric form factors which is consistent with theoretical few body calculations. It involves the variation of the proton charge form factor (size) as a function of proton-nucleon separation. This is not depicted as an average ‘swelling’ of the nucleon, but as a result of short-range dynamics in the proton-nucleon system, as discussed in the next section. We relate the variation of the proton size to the quarks dynamics, here described by the Chromodielectric Soliton Model (CDM).

II. QUARK SUBSTRUCTURE OF NUCLEI AND NUCLEONS

Within the context of soliton models, there have been numerous calculations of nucleon size in nuclear media. Most of these involve immersion of solitons in a uniform (mean) field generated by other nucleons [8]. Another approach has been the 3-quark/6-quark/9-quark bag models, which has been applied to various nuclear properties, including the EMC effect [9]. A hybrid quark-hadron model has been applied by Kisslinger et al. [7] to the He electric form factors with some success.

In a series of papers, Koepf, Pepin, Stancu and Wilets [10–12] have studied the 6-quark substructure of the two-nucleon problem in the framework of the chromodielectric soliton

model [13]. In particular, they calculated the variation of the quark wave functions with inter-nucleon separation. Contrary to previous expectations, the united 6-quark cluster does not exhibit a significant decrease in the quark momentum in spite of an increase in the volume available to the individual quarks [11]. This is due to configuration mixing of higher quark states. Such a momentum decrease had been proffered as an explanation of the EMC effect [14]. However, the united cluster does have approximately twice the confinement volume of each 3-quark cluster, and the quarks extend to a volume nearly three times that of the 3-quark clusters, again enhanced by configuration mixing of excited states.

In Fig. 3 we exhibit the proton form factor based on the calculations of Pepin *et al.* [12] extracted as follows: the soliton-quark structure is a 6-quark deformed composite; the Generator Coordinate Method was used to build a dynamical nucleon-nucleon potential; it involves a Fujiwara transformation, which relates the deformation of the six-quark bag to the effective nucleon-nucleon separation r_{NN} . The proton rms radius is then defined to be

$$r_p = \sqrt{\langle r^2 \rangle - r_{NN}^2/4} \quad (1)$$

where the quark density ρ_q used in calculating $\langle r^2 \rangle = \int \rho_q r^2 d^3r$ has been obtained from 6-quark CDM calculations [13] and normalized to unity. For separated solitons, the r_{NN} is just the separation of the soliton centers and $r_p = 0.83$ fm as indicated by the horizontal line labeled “Free Proton”. Large deformations (near separation) are difficult to calculate. Shown also in the figure is a Gaussian approximation (dashed line) fitted to the CDM result at $r_{NN} = 0$, $r_{NN} = 1$ fm, and in the asymptotic region.

To obtain the latter approximation, we assume a Gaussian form for the variable “proton” charge density. Then the charge distribution due to two nucleons is

$$\begin{aligned} \rho_{pair}(\mathbf{r}_i, \mathbf{r}_j; \mathbf{r}) = & \left\{ \delta_{ip} \exp[-|\mathbf{r} - \mathbf{r}_i|^2/b^2(r_{ij})] \right. \\ & \left. + \delta_{jp} \exp[-|\mathbf{r} - \mathbf{r}_j|^2/b^2(r_{ij})] \right\} / \pi^{3/2} b^3(r_{ij}) \end{aligned} \quad (2)$$

where we indicate explicitly that b is a function of the distance r_{ij} between the nucleons i and j . Here “ p ” stands for “proton” and the Kronecker symbols pick out protons among i and j .

We allow the Gaussian size b to depend on the internucleon distance according to

$$b(r') = b_0[1 + Ae^{-r'^2/s^2}], \quad (3)$$

with the free proton value, $b_0 = \sqrt{2/3} \, 0.83 \text{ fm}$. The Gaussian fit shown in Fig. 3 corresponds to $A = 0.45$ and $s = 1.92 \text{ fm}$.

Using the Independent Pair Approximation (IPA) and Eq. (2), we can calculate the charge distribution from the above result by employing a two-body correlation function, $\rho_2(\mathbf{r}_i, \mathbf{r}_j)$,

$$\rho_{ch}(r) = \sum_{i < j} \int d^3 r_i \int d^3 r_j \rho_2(\mathbf{r}_i, \mathbf{r}_j) \rho_{pair}(\mathbf{r}_i, \mathbf{r}_j; \mathbf{r}) / 3. \quad (4)$$

There are six pairs (i,j) and each “proton” appears three times, hence the factor 1/3. The two-body correlation function is generated from the nuclear wave function (described in the next section) by taking the average

$$\rho_2(\mathbf{r}_1, \mathbf{r}_2) = \int |\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)|^2 d^3 r_3 d^3 r_4, \quad (5)$$

with the constraint $\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4 = \mathbf{0}$.

III. ^4He CHARGE DISTRIBUTION : RESULTS

The ^4He wave function $\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)$ has been obtained by solving the non-relativistic Schrödinger equation :

$$\left[\sum_i -\frac{\hbar^2}{2m} \nabla_i^2 + \sum_{i < j} V_{ij} + \sum_{i < j < k} V_{ijk} \right] \psi = E\psi. \quad (6)$$

where V_{ij} and V_{ijk} are respectively the v_{18} Argonne nucleon-nucleon interaction and the Urbana 9 three-nucleon interaction. The parameters entering V_{ij} have been fitted to the deuteron and two-nucleon scattering data. The parameters of V_{ijk} were determined by fitting the binding energy of $A = 3$ nuclei. The Schrödinger equation (6) has been solved by using the Green’s Function Monte Carlo (GFMC) method, which has proven to be very valuable in studying light nuclei, and produced more accurate results than the so-called Variational Monte Carlo (VMC) method. A typical difference, important for the present study, is that within 0.5 fm of the centre-of-mass the GFMC point density has a slight hole which does not appear in VMC results. Details about the GFMC method and the form of

the interaction potentials used are reviewed in Ref. [5]. The combination of the potentials V_{ij} and V_{ijk} introduced above gives the correct binding energy and approximately the correct rms radius for ${}^4\text{He}$. Previous calculations of other properties of ${}^4\text{He}$ have been done with an older nucleon-nucleon interaction v_{14} [15].

We have calculated the charge distribution for ${}^4\text{He}$ in the spirit of the Independent Pair Approximation using Eq. (4). The proton size parameter $b(\mathbf{r}_1 - \mathbf{r}_2)$ given by Eq. (3) was taken from the Gaussian fit to the calculations of Pepin *et al.* [12] presented in Fig. (3). The resulting distribution is shown in Fig. (4) (solid line). The improvement over the free proton case, i.e., a Gaussian with a constant $b = b_0$ (dotted line) is impressive and leads to a fairly good agreement with the data. In the same figure we also indicate the point density (dashed line) derived straightforwardly from the ${}^4\text{He}$ wave function described above. Even if this density shows a central depression, it is not sufficient by itself to reproduce the data, when combined with the free proton form factor.

IV. CONCLUSIONS

We succeeded in reproducing fairly well the ${}^4\text{He}$ charge distribution by assuming a proton size which increases with increasing density.

We have identified the origin of the variable proton size through the structure function obtained from dynamical 6-quark N-N studies in the spirit of the Independent Pair Approximation.

We could probably improve our results if we recalculate meson effects using the quark structure functions given (say) by the six-quark IPA model. This item is a topic for further investigation.

In addition, one must study the predictions of such models for quasi-elastic scattering. In the quasi-free regime, nucleons models produce a good description of the data as long as realistic nucleon interactions, including charge exchange, are incorporated in the final-state interaction [16]. Unlike the charge form factor, two-body charge operators are expected to play a much smaller role here, principally because this is the dominant channel and there is little interference. The combination of the two regimes provides a critical test for models of structure and dynamics in light nuclei.

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FIGURES

FIG. 1. Model-independent charge distributions for ${}^3\text{He}$ and ${}^4\text{He}$ extracted from experiment. Reproduced from McCarthy *et al.*[1]. , who state that “the extreme limits of $\rho(r)$ cover the statistical, systematical as well as the completeness error of the data.”

FIG. 2. Point-proton density distribution for ${}^4\text{He}$ obtained by unfolding the free proton form factor, allowing for meson exchange corrections and relativistic effects. Reproduced from Sick[2].

FIG. 3. Proton rms charge radius r_p of Eq. (1) as a function of inter-nucleon separation. The line labeled CDM is the calculated chromodielectric model result. The dashed line is a Gaussian approximation, normalized to the free value, with a size parameter given by Eq. (3)

FIG. 4. ${}^4\text{He}$ density distributions: The dashed line is the point density from a parameterized Green’s Function Monte Carlo calculation. The curve labeled “free proton” is the charge distribution obtained from a Gaussian proton charge distribution with a fixed size parameter (as is usually done). The curve labeled “variable proton size” uses the Gaussian fit of Fig. 3. We also indicate half the normal nuclear density $0.17/2 \text{ fm}^{-3}$.

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